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Stability of Dynamical System Using Reduction Technique

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Abstract— In this paper, Dynamic stability analysis of a large interconnected power system is extremely time consuming and laborious and may even exceed the storage capacity of modern fast computers because of the high order of the system matrix. The complexity often makes it difficult to obtain a good understanding of the behavior of a system. The exact analysis of most of the high order system is both tedious and costly; it poses a great challenge to both system analyst and control engineer. So we will use both methods Routh Stability Array (RSA) Method and Stability Equation (SE) Method. To reduce a high-order system, the proposed methods ensures the stability of the reduced system model, if the original high order system is stable.

Index Terms— Reduction Technique, Dynamical System, Linear Time Invariant (LTI), High-order System (HOS), Model Order Reduction (MOR), Single-Input Single-output (SISO) System, Routh Stability Array (RSA) Method, Stability Equation (SE) Method.

1 INTRODUCTION

Nowadays, system becomes very complicated, tedious and costly by increases the use of some external controlling devices and the calculation, complexity, cost becomes major factor and also creates a problem for analyst or programmer to solve those high-order system (HOS) problems [1, 2]. The dynamic stable systems with least cost, minimum complexity, better characteristics, ease in operation, and low time computation requirements are known as the most economical system. Several techniques have been considered for reduction of high-order LTI models into the lower-order models to reduced system complexity, cost-effectiveness as compared to the original system [3]. All the control system and power system networks defined in MATLAB using a block diagram in SIMULINK portion may represent a higher-order transfer function for that system [4]. This transfer function for the simplicity or for ease of operation must be reduced to a lower-order transfer function using a reduction technique prevalent in literature such as Routh approximation [5], Pade approximation [6], Routh-Pade method [7,8], Stability equation method [9-12], Differentiation method [2, 3, 4, 13], Routh Stability array method [10, 14], The proposed methods guarantee the stability of the reduced system model, if the original system of high order is stable.your paper.

2 STATEMENT OF THE PROBLEM

Consider a high order transfer function of a system represented as in Eq.(1)

$$G(s) = \frac{\sum_{i=0}^{n-1} b_i s^i}{\sum_{i=0}^{n} a_i s^i} \qquad (1)$$

where, the G(s) represents a high order system with the order of n The purpose of manuscript is to reduce the order of such high order system to r. The reduced order model may be represented as in Eq.(2).

$$R(s) = \frac{\sum_{j=0}^{n-1} d_j s^i}{\sum_{j=0}^{n} c_j s^i} \qquad (2)$$

Where, ai, bi, cj and dj are the scalar constants of original high order system and the reduced order system. The objective is to find reduced.

3 PROPOSED METHODS

3.1 Routh Stability Array

3.1.1 Routh's Algorithm

This method uses the generation of Routh array by using coefficients of given n[th] high-order polynomial of a problem. First, two rows indicate generated rows, having the coefficients of original HOS. After that all the rows known as computed rows derived from previous

$$P(s) = a_0 s^n + b_0 s^{n-1} + a_1 s^{n-2} + b_1 s^{n-3} + \cdot\ \cdot\ \cdot \qquad (3)$$

Generating rows:

$$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_5 \quad - -$$
$$b_0 \quad b_1 \quad b_2 \quad b_3 \quad - - -$$

Computed Rows:

$$c_0 \quad c_1 \quad c_2 \quad c_3 \; - - -$$
$$d_0 \quad d_1 \quad d_2 \quad - - - -$$

The third and each subsequent row is evaluated from the preceding two rows by means of a systematic form of calculations viz.

$$\left. c_0 = \frac{b_0 a_1 - a_0 b_1}{b_0} \qquad c_1 = \frac{b_0 a_0 - b_0 a_2}{b_0} \right\} \qquad (4)$$

$$d_0 = \frac{c_0 b_1 - b_0 c_0}{c_0} \qquad c_0 = \frac{c_0 b_2 - b_0 c_2}{c_0}$$

There may be a possibility that certain elements in the array may vanish, unless a first column number c_0, d_0 etc. vanishes the computation proceeds without difficulty. The array becomes roughly triangular and terminates with a row having only one element. The number a_0, a_1 . . . b_0, b_1 etc., in the generating rows are normally coefficients taken either alternately from one polynomial or taken from two polynomials. The source of which depends upon the specific application. Routh's criterion is an efficient test that tells how many roots of a polynomial lie in the right half of *s*-plane and also gives information about roots symmetrically located about the origin. The numbers of sign changes in the first column of Routh array determine the number of roots lying in the right of *s*-plane.

Table 1

Routh Array from Numerator.

b_{11}	b_{12}	b_{13}	b_{14}
b_{21}	b_{22}	b_{23}	b_{24}
b_{31}	b_{32}	b_{33}	-
b_{41}	b_{42}	b_{43}	-
. . .			
$b_{m,1}$			
$b_{m+1.1}$			

Table 2

Routh Array from Denominator.

a_{11}	a_{12}	a_{13}	a_{14}
a_{21}	a_{22}	$_{23}$	a_{24}
a_{31}	a_{32}	a_{33}	-
a_{41}	a_{42}	a_{43}	-
. .			
$a_{n,1}$			
$a_{n+1,1}$			

3.1.2 Reduction Algorithm

Consider a higher order system transfer function G(s) is represented by Eq. (5). In the form of numerator and denominator.

$$G(s) = \frac{N_{HOS}(s)}{D_{HOS}(s)} = \frac{b_{11}s^m + b_{21}s^{m-1} + b_{12}s^{m-2} + b_{22}s^{m-3} + \cdots}{a_{11}s^n + a_{21}s^{n-1} + a_{12}s^{n-2} + a_{22}s^{n-3} + \cdots} \qquad (5)$$

Where $a_i\, and\, b_i$ are constants for *i=1,2,…,n*.

The reduced order transfer function is constructed directly from elements in the Routh Stability arrays of the high order numerator and denominator as in Tables (1) and (2).

ROM The reduced model is less than n. The reduced system model in Eq.5 is the model shown in Eq.6. In general, which it retains the original system specifications.

$$G_r(s) = \frac{N_{ROS}(s)}{D_{ROS}(s)} = \frac{c_1 s^{r-1} + c_2 s^{r-2} + \cdots + c_r}{s^r + d_1 s^{r-1} d_2 s^{r-2} + \cdots + d_r} \qquad (6)$$

Where $c_i\, and\, d_i$ are constants for *i=1,2,…,r*.

3.2. Stability Equation Method

3.2.1 Steps for Stability Equation Method and Reduction

In this technique, the transfer function of reduced orders are obtained directly from the pole zero patterns of the stability equations of the n original transfer function of system Thus order of the stability equations of transfer function can be reduced. Assuming, a high order transfer function the system is as shown in Eq.7.

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_{11}s^m + b_{21}s^{m-1} + b_{12}s^{m-2} + b_{22}s^{m-3} + \cdots}{a_{11}s^n + a_{21}s^{n-1} + a_{12}s^{n-2} + a_{22}s^{n-3} + \cdots} \qquad (7)$$

The *N*(*s*) and *D*(*s*) are the numerator and the denominator of *G*(*s*), respectively. The order of *D*(*s*) is *n* and the order of *N*(*s*) is m such that the *n* > *m*.

Step1: Separation of even, odd parts:

The numerator and the denominator of Eq. (7). are separated in even and odd parts. If the even and odd parts are sub-scripted by e and o, respectively. The even and odd polynomials of numerator may be represented by $N_e(s)$ and $N_o(s)$, respectively. Similarly, the even and odd polynomials of denominator may be represented by $D_e(s)$ and $D_o(s)$, respectively. Therefore, the system in Eq.(7) may be represented as in Eq.(8):

$$G(s) = \frac{N_e(s) + N_o(s)}{D_e(s) + D_o(s)} \qquad (8)$$

Where:

$$G(s) = \frac{\sum_{0,2,4}^{m} b_i s^i + \sum_{1,3,5}^{m} b_i s^i}{\sum_{0,2,4}^{n} b_i s^i + \sum_{0,2,4}^{n} b_i s^i} \qquad (9)$$

$$N_e(s) = \sum_{0,2,4}^{m} b_i s^i \qquad (10)$$

$$N_o(s) = \sum_{1,3,5}^{m} b_i s^i$$

$$D_e(s) = \sum_{0,2,4}^{n} b_i s^i$$

$$D_O(s) = \sum_{1,3,5}^{n} b_i s^i$$

The roots of $N_e(s)$ and $N_o(s)$ are called zeros z_i(s) and that of $D_e(s)$and $D_O(s)$are called poles p_i(s).

In this method, a polynomial is reduced by successively discarding the less significant factors.

Let us illustrate the method by reducing the denominator, the numerator is reduced similarly and the ratio of reduced numerator and denominator gives the reduced order model. The denominator is separated as in Eq.11.

$$D(s) = D_e(s) + D_o(s) \qquad (11)$$

Where,

$$De(s) = a_0 + a_2 s^2 + a_4 s^4 + \qquad (12)$$

$$Do(s) = a_1 + a_3 s^3 + a_5 s^5 + \_\,\_\,\_$$

Step2: The Eq. (2.15) may be written as in Eq. (2.16):

$$\left.\begin{aligned} D_e(s) &= a_0 \prod_{i=1}^{m_1}\left(1+\frac{s^2}{z_i^2}\right) \\ D_o(s) &= a_1 s \prod_{i=1}^{m_2}\left(1+\frac{s^2}{z_i^2}\right) \end{aligned}\right\} \qquad (13)$$

Where m_1 and m_2 are the integer parts of

$$\frac{n}{2} \; and \; \frac{n-1}{2} \text{ respectively.}$$

$$\text{and } z_1^2 < p_1^2 < z_2^2 < p_2^2 \ldots$$

Step 3: Dispose of agents with small amounts of $z_i^2 \, and \, p_i^2$ in Eq.(13) . the stability equations for rth order are obtained as in Eq.14.

$$\left.\begin{aligned} D_{er}(s) &= a_0 \prod_{i=1}^{m_3}\left(1+\frac{s^2}{z_i^2}\right) \\ D_{or}(s) &= a_1 s \prod_{i=1}^{m_4}\left(1+\frac{s^2}{z_i^2}\right) \end{aligned}\right\} \qquad (14)$$

Where m_3 and m_4 are the integer parts of

$$\frac{r}{2} \; and \; \frac{r-1}{2} \text{ respectively.}$$

Then we collect the equations in equation (14) and get the new denominator in Eq.15.

$$D_r = D_{er}(s) + D_{or}(s) = \sum_{i=0}^{r} a_{1i} s^r \qquad (15)$$

Therefore, the denominator polynomial in Eq. (2.10) is now known, which is given by Eq.(16)

$$D_r(s) = a_{10} + a_{11}s + a_{12}s^2 + \cdots + a_{1(r-1)}s^{r-1} + a_{1r}s^r \qquad (16)$$

Step 4: In the same way, the numerator can be reduced to the system Eq.(7) We get a new numerator as in the Eq.(17)

$$N_r(s) = N_{er}(s) + N_{or}(s) \qquad (17)$$

Therefore, the numerator polynomial in Eq. (7) is now known, which is given by Eq.(18).

$$N_r(s) = b_{10} + b_{11}s + b_{12}s^2 + \cdots + b_{1(r-1)}s^{r-1} + b_{1r}s^r \qquad (18)$$

The complete model of the rth order reduced model may be represented as in Eq.19.

$$R(s) = \frac{N_{er}(s)+N_{or}(s)}{D_{er}(s)+D_{or}(s)} \qquad (19)$$

3.3 Numerical examples

Consider the HOM (8th order system) as shown in Eq.(20)

$$G(s)=\frac{18s^7+514s^6+5928s^5+36380s^4+122664s^3+222088s^2+185760s+40320}{s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+185760s+40320} \qquad (20)$$

3.3.1 Solution by RSA method

The RSA method applied on given problem shown in Eq. 20. on the basis of Routh stability array for both numerator in Table 3 and denominator in Table 4 computed terms gives the result as shown in Eq. 21.

Table 3.
Routh stability array for numerator

S^7	18	5928	122664	18576
S^6	514	36380	22208	40320
S^5	4653.99	114886.5	184348.01	0
S^4	23691	201728.07	40320	
S^3	75257.9	176427.3	0	
S^2	146109.1	40320		
S^1	155670.6	0		
S^0	40320			

Table 4.
Routh stability array for denominator

S^8	1	546	4536	18124	40320
S^7	36	1	67284	185760	0
S^6	420	380	112964	40320	
S^5	4503.42	57601.37	151446.85		
S^4	-4992.07	98839.69	40320		
S^3	146766.11	187820	0		
S^2	105228.15	40320			
S^1	131584.0065				
S^0	40320				

$$R_2(s)=\frac{155670.6s+40320}{105228.15s^2+131584.0065s+40320} \qquad (21)$$

3.3.2 Solution by SE method

The SE method applied on problem shown in Eq. 20, computed terms gives the result as shown in Eq. 22.

$$R_2(s) = \frac{185760s+40320}{118124s^2+109584s+40320} \qquad (22)$$

3.4 The proposed method is Merge RSA and SE TO ROM

The proposed method contains the use of Routh stability array method for numerator and stability equation method determine the reduced order model. The RSA is used for the reduced numerator and SE for the reduced denominator. The numerator using RSA is given by Eq.23. and the denominator using stability equation method is shown in Eq. 24. The reduced second order reduced model is given as in Eq. 25.

$$N(s) = 155670.6s + 40320 \qquad (23)$$

$$D(s) = 118124s^2 + 109584s + 40320 \qquad (24)$$

$$R_2(s) = \frac{155670.6s+40320}{118124s^2+109584s+40320} \qquad (25)$$

4 DISCUSSION AND RESULTS

In this paper, reduction technique to reduce high order model using (RSA) and (SE) techniques for both the SISO systems. The characteristics of original system were preserved. The obtained results are compared with the methods mentioned previously to show their superiority and compare the proposed technique with original to prove similarity of characteristic. The step response of the original system as in Eq. (20) compared in terms of rise-time, settling-time, peak-time and peak as recruited in Table. (1). The step response of these systems are compared in Fig.(1), we find that reduction methods (The proposed method is Merge RSA and SE TO ROM:) are effective methods of reducing the system and maintaining original properties. But when combining the two methods we get a more efficient way to reduce the system. As a result of reducing the ease of dealing with them and the creation of a new model is simpler for a high-order system. While maintaining input and production behavior and maintaining stability.

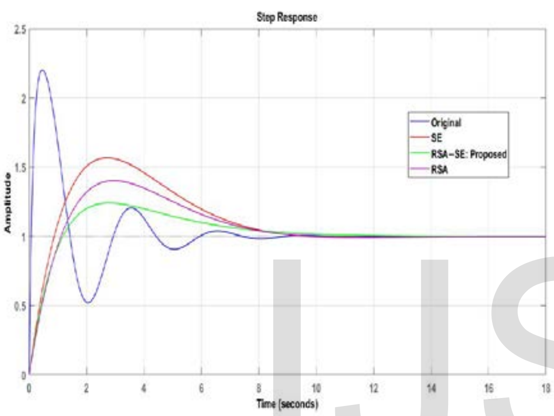

Fig. (1) Comparison of methods

Table 5.

System	Rise Time (s)	Settling Time(s)	Settling Min	Settling Max	Peak	PeakTime
Original (Eq.)	0.0570	7.0067	0.5205	2.2010	2.2010	0.4614
SE-ROM	0.6863	8.7173	0.9367	1.5661	1.5661	2.6806
RSA	0.8648	8.6937	0.9318	1.4030	1.4030	2.9784
RSA and SE	0.8906	9.5344	0.9347	1.2405	1.2405	2.7432

Comparative analysis of step response of original system, methods used in literature and proposed method of system of example

5 Conclusion

In this paper, the system has been these high order system are reduced to 2nd order model using (i) Routh stability array (RSA) method and (ii) the combination of RSA and stability equation (SE) method. The systems reduced order models in literature are considered to compare the response due to proposed RSA based model and proposed RSA-SE based model. The original systems, the respective reduced models of the systems and proposed reduced order models are subjected to step response and its information in terms of rise time, settling time, peak time and peak. The graphical and statistical comparative analysis have been carried out and found to be satisfactory response as compared to original system and reduced models in literature. Applications and extensions.

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